Causality, modal logics and formal logic

Overview of different approaches to Causality

Foundation of "regularity and constant conjunction." Its main proponent is David Hume.

> “Causality is the science of cause and effect. As identifying causal mechanisms is often regarded as a fundamental aspect of most sciences, causality becomes elemental in advancing our knowledge.”[1].

Foundation of 'what if' scenarios of David Lewis.

> Lewis defines causality as, “We think of a cause as something that makes a difference, and the difference it makes must be a difference from what would have happened without it. Had it been absent, its effects—some of them, at least, and usually all—would have been absent as well.”[2]

Involves intervention or manipulation, worded by Michael Sobel.

> “view the cause as an event or state that can be manipulated (or at least potentially manipulated); under this view, causation resides in the existence of a one-to-one correspondence between the state of the manipulated cause and the state of the effect.”[3]

Focuses on the underlying mechanisms of processes, represented by M.Z. Naser.

> “One of the simplest ways to think about mechanisms is to think about processes (that is, biological or chemical processes) with an inherent temporal order.”

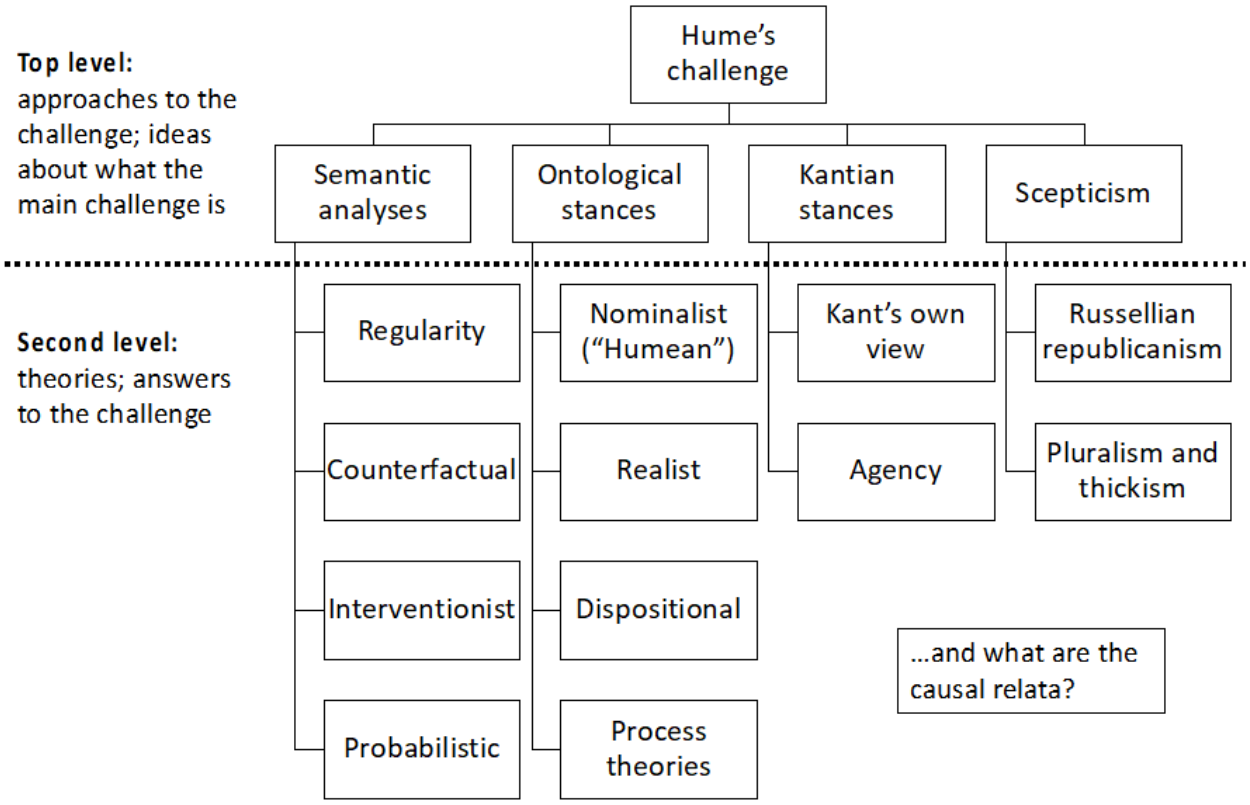
This does not mean that these approaches are mutually exclusive. In fact, it is often rather the opposite. For instance Lewis departed from Hume's view. In the project visualisation of causality, all four approaches proposed by Naser can be integrated without any problem.

In a report about causation Internet Encyclopedia of Philosophy states that all theories about causation find their origine in Hume’s challenge. This illustrated by the diagram below.

1 M.Z. Naser, Simplifying Causality: A Brief Review of Philosophical Views and Definitions with Examples from Economics, Education, Engineering, Medicine, Policy, and Physics, She Ji: The Journal of Design, Economics, and Innovation, Volume 9, Issue 4, 2023, Pages 437-457, ISSN 2405-8726, https://doi.org/10.1016/j.sheji.2024.01.002. (https://www.sciencedirect.com/science/article/pii/S2405872624000029)

2 Idem.

3 Idem.



David Hume's statement, "we may define a cause to be an object followed by another", reflects his view of causality, which emphasises the importance of constant conjunction and temporal priority. This means that for an event to be considered the cause of another, the two events must consistently occur together (constant conjunction) and the cause must precede the effect in time.

Hume's perspective on causality is deeply rooted in his empiricist philosophy, which emphasises experience as the source of knowledge. He argued that our idea of causality is not based on an inherent necessity in the objects themselves, but rather on our repeated observation of events that follow each other regularly.

Modal logics versus formal logics

The fundamental investigation of causality is the field of modal logic. However, there are some important differences between modal logic and formal logic. Since mathematical formal logic receives much more attention in education than modal logic, it is useful to briefly review those differences.

(1) A causal relation implies a logic implication but not the other way around

$(p \ \Box\!\!\rightarrow \ q) \rightarrow (p \rightarrow q)$

This reads as "if q is a necessary consequence of p" then p, implies q".

(2) While formal logics claims to be timeless, modal logics depends on time series. The cause preceeds the consequence.

(3) Modal logics does not use truts tables but decision-making procedures.

(4) Causation can be represented by a directed graph.

> “Our basic idea is simply this: we describe properties of directed graphs consisting of points (‘possible worlds’ if you like grandeur) with directed links encoded in an ‘accessibility relation’ between points. A universal modality □ Ø is true at a point in a graph if Ø is true at all points reachable by a directed arrow.”[4]

(5) The most important decision-making procedure for causation is the ‘counterfactual’ test. Schürman definined the test for the‘Why-Because’ analysis (WBA).

> “To check the correctness of a cause-and-effect relationship, the Counterfactual Test (CT), based on work of David Lewis and David Hume, is used. "If the (potential) causal factor had not occurred, could the effect have occurred?" If this test is answered with "no", then the potential causal factor is a "necessary causal factor" (abbr. NCF). Use of the CT ensures that all nodes in the WBG are correctly linked.” [5]

(6) In Frege's mathematical logic, an argument does not require that the premises are actually true, but rather that the premises, if true, would guarantee the truth of the argument's conclusion. For the sake of the 'counterfactual' test, this does not apply to causation. Cause and effect must both be observably true to prove causation.

(7) While implication is transitive, transitivity in causation is contested. In classical logic, if X implies Y and Y implies Z then X implies Z as well.

Transitivity revisited

David Lewis founder of modern modal logic saw causality as transitive. Sara Berstein:

> "Closeness of worlds is to be judged in terms of comparative overall similarity to the actual world based on similarities between those worlds and the actual one. Additionally, so that causation is transitive, causation is the ancestral of counterfactual dependence: there is a string of counterfactual dependencies between c and e."[6]

However, Lewis' 1973 vision was confronted with clear counterexamples in different types of redundant causality, or cases where there are multiple sufficient causes to bring about an outcome[7]. In modal logic, there is an additional dependent relation between cause and effect, the 'conterfactual dependence'. If Z is not also contrafactually dependent on X, there is no transitivity according to Beckers and Vennekens[8]. As to the researchers all examples of non-transitivity can be reduces to three fenomena: ‘symetric overdetermination’, ‘early preemption’ and ‘late preemtion’ [9].

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4 van Benthem, Johan, (IEP), Modal Logic: A Contemporary View, University of Amsterdam, Stanford University, and Tsinghua University, The Netherlands, U. S. A., and China, <https://iep.utm.edu/modal-lo/>.

5 Schürmann, Tim, (WBA) 'Counterfactual Test’, Workgroup RVS, Faculty of Technology, Bielefeld University) <https://rvs-bi.de/research/WBA/IntroWBA-ENG.pdf>.

6 Bernstein, Sara. (2019). Lewis’s Theories of Causation and Their Influence. 10.1017/9781316779651.015., <https://sarajbernstein.github.io/sjb/LewisTCTI.pdf>.

7 Menzies, Peter and Helen Beebee, "Counterfactual Theories of Causation", The Stanford Encyclopedia of Philosophy (Fall 2025 Edition), Edward N. Zalta & Uri Nodelman (eds.), forthcoming URL = <https://plato.stanford.edu/archives/fall2025/entries/causation-counterfactual/>.

8 Sander Beckers and Joost Vennekens, 2017, The Transitivity and Asymmetry of Actual Causation, Ergo: An Open Access Journal of Philosophy, volume 4, pp 1-27. <https://quod.lib.umich.edu/e/ergo/12405314.0004.001/--transitivity-and-asymmetry-of-actual-causation?rgn=main;view=fulltext>.

9 Idem

An example to clarify the dependence of the 'counterfactual' test. The formation of rain clouds depends on the presence atmospheric condensate of water vapour and ice crystals above the zero degree boundary. So you have to have clouds to form rain clouds. The rain falls through the rain cloud base after droplets of 20 microns have grown to 2000 microns due to collisions during the up and down motion of droplets and ice crystals. So the rain depends on the presence of those clouds. So we have unambiguous transivity here. If those clouds had not been there, it would not have rained.

Another problem may be that an obvious common cause of all processes is not mentioned. When it comes to processes involving living things, survival instinct will often play an important role. **In all cases of doubt about transitivity, a "dotted line" instead of a "solid line" can be used.**